Exam: Optics and Optical design

Instructions: The five problems are not ordered by increasing difficulty. For each problem a maximum of six points will be given. Fifteen points are required for passing the exam. Observe that solutions without reasonable motivation are not acceptable, even if the final answer is correct.
You are allowed to bring: For the first problem: no help
For the other: Calculator, TEFYMA, the course book.

1. a) Explain the differences between homogeneous, isotropic, and dispersive media.
   b) Large beam splitters in the shape of windows are used in supermarkets to supervise the customers or by the police in interrogation rooms. Explain why the clerk in the shop behind the window can see you, but you cannot see him/her. (Tip: It has nothing to with polarization. In principle, any window can do, if the lighting is chosen correctly.)

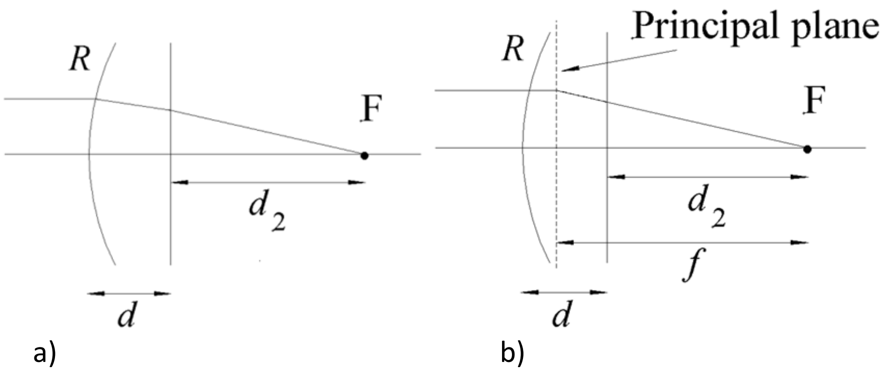

Figure 1. a) Imaging with a thick lens. b) Equivalent ray diagram.

2. A thick convex-plane lens with a radius R and refractive index n is surrounded by air according to figure 1a.
   a) Determine the ray-transfer matrix of the system between the first surface and the focal plane.
   b) Give a formula for the distance d_2 as a function of R n and d.
   The thick lens can be replaced with a plane (principal plane) where all the refractions occur (figure 1b). Then the focal point F is located at the distance f from this plane.
   c) Show that the distance f of the lens is given by the formula:
   $$f = \frac{R}{n-1}$$
   d) Show that this is the focal length of the lens when it can be assumed to be a thin lens.
   A house with the height 8 m and at a distance of 2.4 km is imaged with a thin convex plane lens with R = 12 cm and n = 1.5 onto a CCD sensor that has 7.68 million square pixels on an area of 8 mm x 6 mm.
   What is the number of CCD-pixels covered from the bottom to the top of the image?

3. A monochromatic plane wave with wavelength λ=600 nm illuminates a circular aperture with D=1 mm diameter. The resulting diffraction pattern is observed on a screen in the distance d=10 m (see Fig. 2a). The screen can be considered to be in the far-field (Fraunhofer diffraction).

a) Describe the diffraction pattern observed on the screen. Calculate the radius of the central disk.

b) We replace the aperture with another one (Fig. 2b), consisting of two identical holes with diameter D=1 mm each, separated from each other by a distance a=2 mm (center-to-center). The observed diffraction pattern is an interference of the individual ones. Describe the diffraction pattern. Calculate the period of the interference fringes.

c) We now put a very thin film on one of the holes, which delays the light by half a wavelength. Explain what happens to the interference fringes.

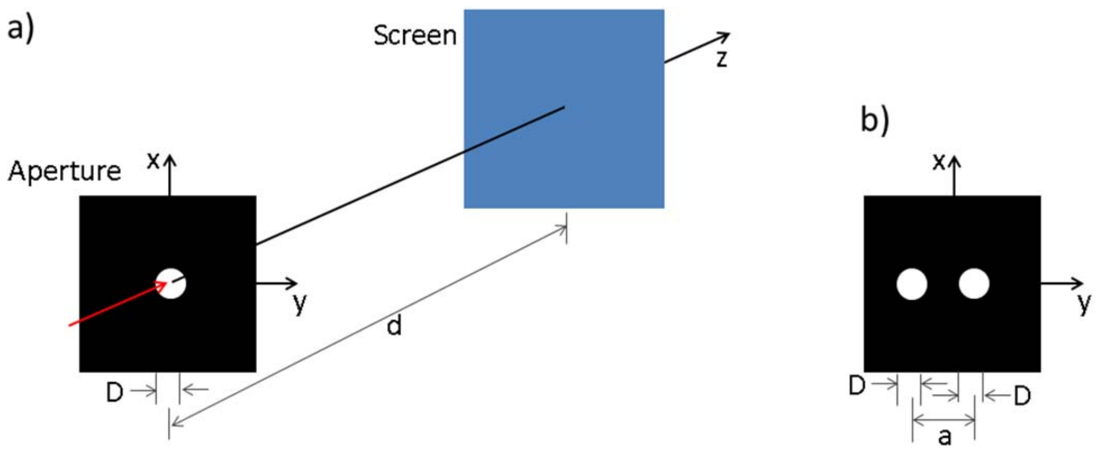

Figure 2: Two different apertures generate diffraction patterns on a screen placed at distance d in the far-field.

4. We look at the so-called Fresnel rhomb (see Fig. 3a). The Fresnel rhomb is a prism, which was designed by Augustin-Jean Fresnel in 1817 as an extremely achromatic λ/4-retarder.

a) Explain how the Fresnel rhomb can make circularly polarized light from linearly polarized and vice versa. How must the incoming linear polarization be oriented in order to get circular polarization out?

b) Calculate the transmission of the device, taking into account that the input and exit faces are not anti-reflection coated. The refractive indices of the glass and air are n_g=1.51 and n_a=1, respectively.

c) The phase-shift $\varphi(\theta_i) = \varphi_{TE}(\theta_i) - \varphi_{TM}(\theta_i)$ between the TE- and TM-components in total internal reflection as function of the incoming/reflection angle can be written as:

$$tan\left(\frac{\varphi}{2}\right) = \frac{cos(\theta_i)\sqrt{sin^2(\theta_i) - \frac{n_a^2}{n_g^2}}}{sin^2(\theta_i)}$$

Note that there is a sign error in FoP (6.2-13). The phase-shift φ is plotted in Fig. 2b. Find the two angles θ_i that make the Fresnel rhomb a λ/4-plate.

d) Due to their broad bandwidth achromatic wave plates are very important for ultrashort laser pulses. A pulse of 10 fs duration has a bandwidth of approximately σ_λ=100 nm centred around 800 nm. Typical glass used for a Fresnel rhomb has a dispersion coefficient of $D_\nu \approx 3.1 \times 10^{-25}$ s^2/m. Calculate the pulse duration after passing the Fresnel rhomb and argue, if the device is suited for ultrashort pulses. (Tip: take an angle θ_i from sub-problem c) or estimate one from Fig. 3b.)

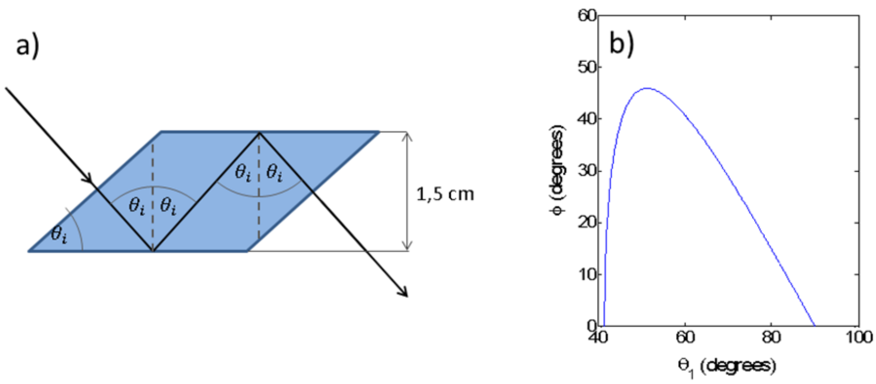

Figure 3: a) Fresnel rhomb. b) Phase shift in total internal reflection between TE- and TM-polarization.

5. As we learned in problem 4, achromatic wave retarders are very important for ultrashort laser pulses with large bandwidth. Now, we want to look at the difference between so-called *zero-order* and *multi-order* wave retarders. A zero-order wave retarder retards exactly $\lambda/4$ for a $\lambda/4$-plate, whereas a $\lambda/4$-plate of order five retards by 21 times $\lambda/4$ ($5\lambda+\lambda/4$). True zero-order wave retarders are often so thin that they are either very complicated to produce or very delicate to handle. For that reason multi-order retarders are often used.
Here, the wave retarders are made of crystalline quartz with the refractive indices n_o=1.5394 and n_e=1.5483 for a vacuum wavelength of λ=800 nm.

a) Calculate the thickness of a zero-order $\lambda/4$ wave retarder and of one of order five.

b) One can get an idea about how achromatic the above wave retarders are by calculating the closest wavelength, at which the retarder becomes a $\lambda/2$-plate. Compare the two plates.

c) We now look at an application of a $\lambda/4$-retarder. Circular polarization (right or left) is incident on a $\lambda/4$-retarder, which is followed by a polarizer. Using the Jones vector formalism find the orientation angle θ between the retarder and the polarizer that maximizes the throughput through the polarizer. What is the maximum throughput?

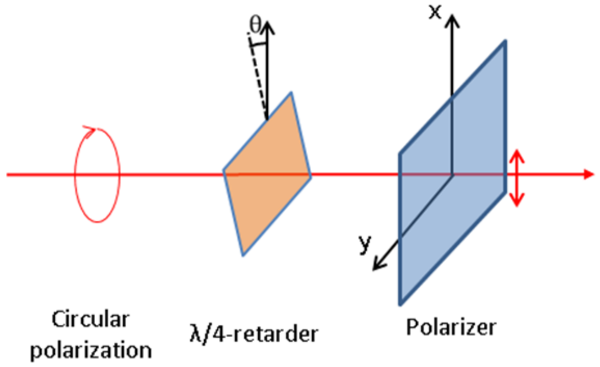

Figure 4: Incoming circular polarization hits a $\lambda/4$-retarder, which is rotated an angle θ from the polarizer on the right.